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An upper bound on the total cross-section for electroweak baryon number violation

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Abstract: An upper bound on the total cross-section of s-wave electroweak instanton/sphaleron induced baryon plus lepton number violating processes is presented. It is obtained by exploiting a recently reported lower bound on the corresponding tunneling suppression exponent and by estimating the pre-exponential factor. We find that the present knowledge about electroweak baryon plus lepton number violating processes still allows their eventual observability at the Very Large Hadron Collider, even as pure s-wave scattering. An observable rate at cosmic ray facilities and neutrino telescopes from the s-wave is excluded. It would require a substantial contribution from higher partial waves.

Keywords: Nonperturbative Effects, Solitons Monopoles and Instantons, Standard Model.

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Contents

1. Introduction

Baryon and lepton number are not strictly conserved in the standard electroweak model [1]. In the background of topological non-trivial fluctuations of SU(2) gauge fields with topological charge $Q \neq 0$, notably of instantons ($Q = 1$) and anti-instantons ($Q = -1$) [2], baryon (B) and lepton (L) number change according to $\triangle B = \triangle L = -n_{\rm gen}\, Q$, where $n_{\rm gen} = 3$ is the number of fermion generations. Such fluctuations correspond to transitions between degenerate, topologically inequivalent vacua [3]. The latter are known to be separated by energy barriers whose minimum height is given by the static energy $E_{\rm sp}$ of an unstable static solution of the classical Yang-Mills equations called the sphaleron [4]. Its value,[1] $E_{\rm sp} \approx \pi\, m_W/\alpha_W \simeq 7.5\,{\rm TeV}$, sets the scale for non-perturbative $B + L$ violation in the Standard Model. Indeed, while these processes are exponentially suppressed at energies or temperatures below the sphaleron energy by a tunneling factor, they are known to have a sizeable rate for temperatures above $E_{\rm sp}$ [6] and to have a crucial impact on the evolution of the baryon asymmetry of the universe. On the other hand, the long-standing question, first raised in refs. [7, 8], whether these processes occur with sizeable rates in high energy particle collisions, is still not finally settled (for reviews, see ref. [9]).

It is the purpose of this Letter to elaborate and to compare recent results on this issue [10]–[12] and to work out an upper bound on the total cross-section for electroweak baryon number violating processes. We will show that the present knowledge about the latter still allows their eventual observability at future colliders [11, 13], such as the Very Large Hadron Collider (VLHC) [14]. We will also discuss implications for searches at cosmic ray facilities and neutrino telescopes [15, 16].

[1] Here, $m_W = 80.423(39)$ GeV is the $W^{\pm}$ boson mass and $\alpha_W(m_W) = 0.033819(23)$ is the SU(2) fine structure constant [5].

2. Setting the stage

At center-of-mass (CM) energies much less than the sphaleron energy, the rates of anomalous electroweak $B+L$ violation are rapidly growing [8]. The corresponding total cross-section is known to have an exponential form [17]. Including essential pre-exponential factors [18], one has, for the phenomenologically interesting case of fermion-fermion scattering via electroweak instantons/sphalerons, $\mathrm{f}+\mathrm{f} \xrightarrow{I}$ all,

$$\begin{aligned}\hat{\sigma}_{\rm ff}^{(I)} &\approx \frac{1}{m_W^2}\left(\frac{2\pi}{\alpha_W}\right)^{7/2} \exp\left[-\frac{4\pi}{\alpha_W}\,F_W\left(\frac{\sqrt{\hat{s}}}{4\pi m_W/\alpha_W}\right)\right] \\ &\simeq 5.3\times 10^3\ \text{mb}\ \exp\left[-\frac{4\pi}{\alpha_W}\,F_W\left(\frac{\sqrt{\hat{s}}}{4\pi m_W/\alpha_W}\right)\right], \end{aligned} \tag{2.1}$$

where $\sqrt{\hat{s}}$ denotes the fermion-fermion CM energy. By means of perturbative calculations of the relevant exclusive amplitudes about the instanton (I), squaring them and summing over the final states, or, alternatively, by means of a perturbative calculation of the forward elastic scattering amplitude about the widely separated instanton anti-instanton ($I\overline{I}$) pair and determining the imaginary part to get the total cross-section via the optical theorem, one may calculate the decisive tunneling suppression exponent F_W, which is sometimes called "holy-grail function" [9], as a series in fractional powers of $\epsilon \equiv \sqrt{\hat{s}}/(4\pi m_W/\alpha_W) \simeq \sqrt{\hat{s}}/(30\,\text{TeV})$ [18, 19],

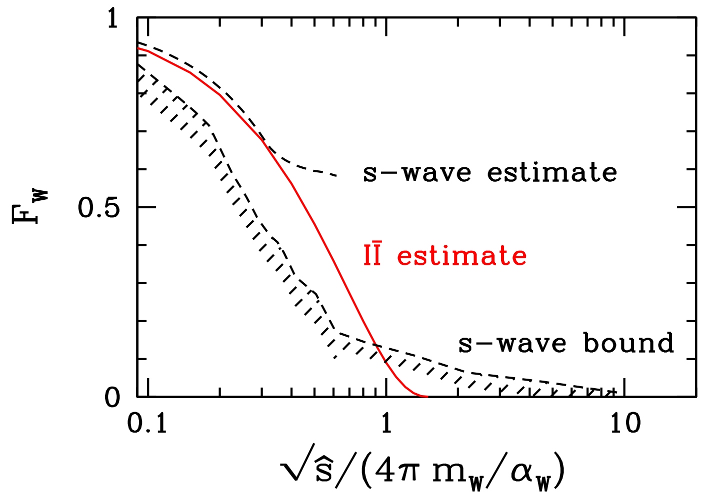
$$F_W(\epsilon) = 1 - \frac{3^{4/3}}{2}\,\epsilon^{4/3} + \frac{3}{2}\,\epsilon^2 + \mathcal{O}\left(\epsilon^{8/3}\right). \tag{2.2}$$

Correspondingly, the total cross-section (2.1) is exponentially growing at $\epsilon \ll 1$. At $\epsilon = \mathcal{O}(1)$, however, the perturbative expression (2.2) ceases to be helpful. In this energy regime, only estimates/extrapolations of and lower bounds on the tunneling suppression exponent exist (cf. figure 1 (top)), which have been quantified recently [10, 11, 12].

3. Estimates

In almost all theoretical investigations of electroweak $B+L$ violation in high energy collisions, the pre-exponential factor in eq. (2.1) is not considered. It is, however, numerically very large and has therefore to be taken into account before any conclusion on the observability of the effect can be drawn. Its size is mainly determined by the large universal factor $(2\pi/\alpha_W)^{7/2} \simeq 8.7\times 10^7$, where the exponent 7/2 can be easily understood as being the result of subtracting from the familiar nominal power $4\,N_c = 8$ [1] the power 9/2 — nine being the effective number of collective coordinate saddle-point integrations [18, 11]. The large pre-exponential factor in eq. (2.1) leads to the fact that a cross-section $\hat{\sigma}_{\rm ff}^{(I)} \gtrsim 10^{-3}$ fb, observable at the projected VLHC[2] [14], requires only a moderate reduction in the tunneling suppression exponent, $F_W \lesssim 0.12$, from its value at zero energy, $F_W(0) = 1$. If a

[2] The VLHC has a projected proton-proton CM energy of $\sqrt{s} = 200\,\text{TeV}$ and a luminosity of about $\mathcal{L} \approx 6\cdot 10^2\,\text{fb}^{-1}\,\text{yr}^{-1}$ [14].



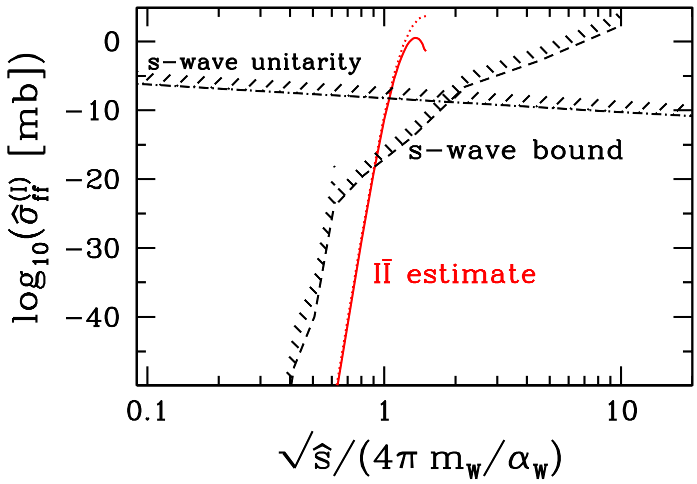


Figure 1: *Top:* The tunneling suppression exponent F_W as estimated from the $I\bar{I}$ interaction [10] (solid). Also shown is its estimate (dashed) and its lower bound (dashed shaded) for s-wave scattering [12]. *Bottom:* The total cross-section $\hat{\sigma}_{\rm ff}^{(I)}$ for electroweak instanton-induced fermion-fermion scattering, $\mathrm{f}+\mathrm{f} \xrightarrow{I}$ all, as estimated from the $I\bar{I}$ interaction, with complete calculation of the pre-exponential factors [11] (solid) and with exploitation of the estimate in eq. (2.1) for the pre-exponential factor (dotted). Also shown is the estimate (dashed) and upper bound (dashed shaded) on the total cross-section for s-wave scattering obtained from eq. (2.1) with exploitation of the estimate and the lower bound on the tunneling suppression exponent for s-wave scattering from ref. [12], respectively. The s-wave unitarity bound (4.1) is displayed also (dashed-dotted shaded).

sizeable reduction in $F_W \lesssim 0.02$ is realized in nature, it is even possible to obtain a large cross-section of hadronic size, $\hat{\sigma}_{\rm ff}^{(I)} \gtrsim 1$ mb, which is observable in present day or near future cosmic ray facilities and neutrino telescopes in the form of cosmic proton or cosmic neutrino initiated events [15, 16].

This numerical fact is clearly demonstrated by the result for $\hat{\sigma}_{\rm ff}^{(I)}$ obtained in ref. [11] (cf. figure 1 (bottom; solid and dotted)), where the tunneling suppression exponent has been estimated via the optical theorem from the $I\bar{I}$ interaction, known in pure SU(N_c) gauge theory for arbitrary separation [10, 20], (cf. figure 1 (top; solid)), and where the complete pre-exponential factor, with inclusion of its energy dependence, has been calculated in the saddle-point approximation. First of all, we see that the approximation of the pre-exponential factor displayed in eq. (2.1), leading to the estimate shown as a dotted line in figure 1 (bottom), gives in fact a reliable estimate of the complete pre-exponential factor over almost the full energy range. Moreover, the estimate of the cross-section becomes indeed of order 10^{-3} fb (1 mb) as soon as the estimate of the tunneling suppression exponent becomes of order 0.12 (0.02).

The analytical estimate based on the $I\overline{I}$ interaction in pure gauge theory is based on a number of assumptions which, at energies above the sphaleron, $\epsilon \gtrsim 0.3$, may or may not be valid. Another method to infer the tunneling suppression exponent was proposed in refs. [21]–[23] and the results of a respective extended quantitative study was presented recently in ref. [12]. The method is based on the observation that the inclusive probability $P(\sqrt{\hat{s}}, n)$ of tunneling from a state with energy $\sqrt{\hat{s}}$ and number of incoming particles n is calculable semi-classically, provided that $\epsilon \equiv \sqrt{\hat{s}}/(4\pi m_W/\alpha_W)$ and $\nu \equiv n\,\alpha_W$ are held fixed in the limit $\alpha_W \to 0$. In this regime, the probability $P(\sqrt{\hat{s}}, n)$ has also an exponential form,

$$P(\sqrt{\hat{s}}, n) \propto \exp\left[-\frac{4\pi}{\alpha_W}\, F_W\,(\epsilon, \nu)\right], \tag{3.1}$$

and the corresponding tunneling suppression exponent $F_W(\epsilon, \nu)$ can be obtained by solving a classical boundary value problem for the Yang-Mills equations. Furthermore, it has been conjectured – and proven in the first few orders of the perturbative expansion in fractional powers of ϵ (cf. eq. (2.2)) [22, 24] as well as by comparison with the full quantum mechanical solution in a model with two degrees of freedom [25] — that the tunneling suppression exponent for the two-particle cross-section is recovered in the limit of a small number of incoming particles,

$$F_W\,(\epsilon) = \lim_{\nu \to 0} F_W\,(\epsilon, \nu)\,. \tag{3.2}$$

In ref. [12], the relevant classical boundary value problem was solved, for spatially spherically symmetric[3] configurations of the SU(2)-Higgs gauge theory, for a large range of ϵ and ν. Though computational limitations did not allow to reach literally the limit $\nu = 0$, the authors were able to extrapolate their results for the tunneling suppression exponent for multiple incoming particles, $F_W\,(\epsilon, \nu)$, to zero ν and to get thereby a stringent lower bound on $F_W(\epsilon)$ (cf. figure 1 (top; dashed shaded)). In addition, they provided an independent estimate for $F_W(\epsilon)$ (cf. figure 1 (top; dashed)).

It is important to note that the lower bound on and the estimate of $F_W(\epsilon)$ obtained in ref. [12] are applicable, strictly speaking, only for s-wave scattering, since they have been obtained exploiting spatially spherically symmetric configurations. This property is shared by the dominant intermediate saddle-point configurations encountered in the $I\overline{I}$ "valley" configuration [10, 20]. Therefore, the estimate of $F_W(\epsilon)$ from ref. [12] should, at small energies, coincide with the analytical estimate based on the $I\overline{I}$ interaction. This is clearly demonstrated in figure 1 (top; dashed and solid, respectively). At higher energies, above

[3]Without this simplification provided by spherical symmetry, the computational cost of the numerical analysis seems to be prohibitive at present.

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the sphaleron, $\epsilon \gtrsim 0.3$, this coincidence is no more observed. Apparently, the estimate based on the $I\bar{I}$ interaction, which has been argued in ref. [11] to be reliable for $\epsilon \lesssim 0.75$, gives a more optimistic value for the two-particle tunneling suppression exponent than the one from the extrapolation of $F_W(\epsilon, \nu)$ to $\nu = 0$.

4. An upper bound on the cross-section

At energies above the sphaleron, $\epsilon \gtrsim 0.3$, the total cross-section of electroweak baryon number violation may well be dominated by axial symmetric configurations [26], on which there is at present no information available. Nevertheless, it is very instructive to exploit the lower bound on the tunneling suppression exponent for s-wave scattering from ref. [12] and to determine from it, via eq. (2.1), an upper bound on the cross-section of s-wave electroweak $B + L$ violating processes initiated by the scattering of two fermions. We find that the estimate based on the $I\bar{I}$ interaction violates this s-wave upper bound for $\epsilon \gtrsim 0.9$ (cf. figure 1 (bottom)). However, as shown in figure 1 (bottom; dashed shaded), the upper bound is rapidly getting less stringent and exceeds $\gtrsim 10^{-3}$ fb for $\epsilon \simeq \sqrt{\hat{s}}/(30\,\mathrm{TeV}) \gtrsim 1.2$. Therefore, it does not exclude the observability of electroweak $B + L$ violation at the VLHC. The upper bound even grows above 1 mb (100 mb) for $\epsilon \gtrsim 7.5$ ($\epsilon \gtrsim 9.7$), thus still allowing for observable consequences of anomalous electroweak processes in ultrahigh energy ($E = s/(2\,m_p) \gtrsim 5\times 10^{19}$ eV) cosmic ray and neutrino physics [15, 16]. However, in order to reach such an $\mathcal{O}$(mb) cross-section, as also suggested by the $I\bar{I}$ estimate in figure 1 (bottom), a substantial contribution from higher partial waves is required. The $I\bar{I}$ estimate and the upper bound violate the s-wave unitarity bound for the inelastic cross-section,

$$\hat{\sigma}_{\mathrm{inel}}^{(\ell=0)} < \frac{4\pi}{\hat{s}} \simeq 5.5\,\mathrm{pb}\,\frac{1}{\epsilon^2}\,, \tag{4.1}$$

above $\epsilon \gtrsim 1.1$ and $\epsilon \gtrsim 1.9$, respectively.

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